Date 24-11-20

PAPER - III

Page No.:
Date : / /

Subgroup

Th A non-empty subset H of a group G is a subgroup of G iff

i) $a, b \in H \Rightarrow ab \in H$

ii) $a \in H \Rightarrow a^{-1} \in H$

Proof: Let H be a subgroup of G. then by definition it follows that (i) & (ii) hold.

Conversely let the given conditions hold in H.

Closure holds in H by i)

Again $a, b, c \in H \Rightarrow a, b, c \in G$

$\Rightarrow a(bc) = (ab)c$.

Hence associativity holds in H.

Also for any $a \in H$, $a^{-1} \in H$ and so by i) $aa^{-1} \in H \Rightarrow e \in H$

thus H has identity

Inverse of each element of H is in H by (ii)

Hence H satisfies all conditions in definition of a group and thus it forms a group and therefore a subgroup of G.